APPENDIX I

MKS UNITS AND DIMENSIONS

Quantity	*Unit*	*Dimensions*
Length	Meter	L
Mass	Kilogram	M
Time	Second	T
Charge	Coulomb	Q
Velocity	Meter /second	LT^{-1}
Force	Newton	LMT^{-2}
Energy, work	Joule	L^2MT^{-2}
Power	Watt	L^2MT^{-3}
Electric potential	Volt	$L^2MT^{-2}Q^{-1}$
Electric field intensity E	Volt /meter	$LMT^{-2}Q^{-1}$
Electric flux	Coulomb	Q
Electric flux density D	Coulomb /square meter	$L^{-2}Q$
Capacity	Farad	$L^{-2}M^{-1}T^2Q^2$
ε_o (permittivity of free space)	Farad /meter	$L^{-3}M^{-1}T^2Q^2$
Electric current	Ampere	$T^{-1}Q$
Magnetic field intensity H	Ampere /meter	$L^{-1}T^{-1}Q$
Magnetic flux	Weber	$L^2MT^{-1}Q^{-1}$
Magnetic flux density	Weber /square meter	$MT^{-1}Q^{-1}$
Inductance	Henry	L^2MQ^{-2}
μ_o(permeability of free space)	Henry /meter	LMQ^{-2}
Electric resistance	Ohm	$L^2MT^{-1}Q^{-2}$

APPENDIX II

TABLE OF PHYSICAL CONSTANTS AND CONVERSION BETWEEN UNITS*

Permittivity of free space	$\varepsilon_o = 8.854 \times 10^{-12}$	farad/meter
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$	henry/meter
Velocity of light	$c = 2.998 \times 10^{8}$	meters/second
Charge of the electron	$e = 1.602 \times 10^{-19}$	coulomb
Mass of the electron	$m = 9.108 \times 10^{-31}$	kilogram
Ratio: electron charge/electron mass	$e/m = 1.759 \times 10^{11}$	coulomb/kilogram
Boltzmann's constant	$k = 1.380 \times 10^{-23}$	joule/degree
Planck's constant	$h = 6.625 \times 10^{-34}$	joule-second

Conversion Between Units

1 angstrom (A) = 10^{-10} meter
1 micron (μ) = 10^{-6} meter
1 gauss = 10^{-4} weber/square meter
1 oersted = $(1/4\pi) \times 10^{3}$ amperes/meter
1 pound = 0.4536 kilogram
1 liter = 1000 cm^3
1 torr = 1 mm of Hg pressure
= 13.595 kilograms/square meter
1 electron volt = 1.602×10^{-19} joule

*For adjusted best values of the physical constants as of 1955, see E. R. Cohen, K. M. Crowe, and J. W. M. Dumond, *Fundamental Constants of Physics*, Interscience Publishers, Inc., New York, 1957.

APPENDIX III

SOME RELATIONSHIPS PERTAINING TO ELECTRIC AND MAGNETIC FIELDS AND CURRENT FLOW

(a) Static Fields

Electric Fields

Electric field intensity . . . . **E**, E
Electric flux density . . . . **D**, D
Electric potential V
Permittivity of free space . . . . ε_o
Relative dielectric constant . . . . ε
Space charge density . . . . ρ
Total charge . . . . q

* $$\mathbf{D} = \varepsilon\varepsilon_o\mathbf{E}$$

$$\int_{\text{closed surface}} \mathbf{D}\cdot\mathbf{n}dS = \int_{\text{volume}} \rho dv = q$$

* $$\nabla\cdot\mathbf{D} = \rho$$

*Energy stored per unit volume $= \frac{1}{2}\varepsilon\varepsilon_o E^2$

$$V_{AB} = -\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

$$\mathbf{E} = -\nabla V$$

In a region of uniform dielectric constant where there is no distributed charge density:

$$\nabla^2 V = 0$$

In the presence of a distributed charge density ρ

$$\nabla^2 V = -\frac{\rho}{\varepsilon_o}$$

*At the interface between two dielectric materials, the normal component of **D** and the tangential component of **E** are continuous. (It is assumed here that there are no surface charges.)

Magnetic Fields

Magnetic field intensity . . . . **H**, H
Magnetic flux density . . . . **B**, B
Magnetic potential . . . . ψ
Permeability of free space . . . . μ_o
Relative permeability . . . . μ
Current density vector . . . . **J**
Total current I

* $$\mathbf{B} = \mu\mu_o\mathbf{H}$$

$$\oint_{\text{closed path}} \mathbf{H}\cdot d\mathbf{l} = I$$

$$\nabla\times\mathbf{H} = \mathbf{J}$$

* $$\nabla\cdot\mathbf{B} = 0$$

*Energy stored per unit volume $= \frac{1}{2}\mu\mu_o H^2$

In the absence of current-carrying conductors:

$$\psi_{AB} = -\int_A^B \mathbf{H}\cdot d\mathbf{l}$$

$$\mathbf{H} = -\nabla\psi$$

In a region of uniform permeability *and in the absence of current carrying conductors:*

$$\nabla^2\psi = 0$$

*At the interface between two magnetic materials, the normal component of **B** and the tangential component of **H** are continuous. (It is assumed here that there are no surface currents flowing at the interface.)

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*These relations also apply to time-varying fields.

(b) Time-Varying Fields

Maxwell's Equations are:

$$\nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t}$$

$$\nabla \times \mathcal{H} = \mathcal{J} + \frac{\partial \mathcal{D}}{\partial t}$$

$$\nabla \cdot \mathcal{D} = \rho$$

$$\nabla \cdot \mathcal{B} = 0$$

Script letters are used here to indicate that the field components are time varying. If the field components vary with a single angular frequency ω, we can set

$$\mathcal{E} = \mathrm{Re}\mathbf{E}\epsilon^{j\omega t}, \qquad \mathcal{B} = \mathrm{Re}\mathbf{B}\epsilon^{j\omega t}, \text{ etc.,}$$

and

$$\frac{\partial \mathcal{B}}{\partial t} = \mathrm{Re}j\omega\mathbf{B}\epsilon^{j\omega t}, \text{ etc.}$$

Then

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

(c) Current Flow

Ohm's Law can be written as

$$\mathbf{J} = \sigma\mathbf{E}$$

or

$$V = IR$$

where σ is the conductivity of the medium, and R is the resistance between the terminals where the voltage V is measured.

The equation of continuity can be written as

$$\nabla \cdot \mathcal{J} = -\frac{\partial \rho}{\partial t}$$

If the current density $\mathcal{J}$ is time varying at a single angular frequency ω, we can set $\mathcal{J} = \mathrm{Re}\mathbf{J}\epsilon^{j\omega t}$. Then

$$\nabla \cdot \mathbf{J} = -j\omega\rho$$

APPENDIX IV

A SUMMARY OF RELATIONS PERTAINING TO THE VELOCITY DISTRIBUTION, ENERGY DISTRIBUTION, AND ANGULAR DISTRIBUTION OF THE ELECTRONS EMITTED FROM A THERMIONIC CATHODE

Symbols:

u_n = velocity of an emitted electron in the direction normal to the emitting surface in meters per second.

u_t = velocity of an emitted electron in the "transverse" direction, or parallel to the emitting surface, in meters per second.

u = total emission velocity in meters per second.

W_n = kinetic energy of an emitted electron in the direction normal to the emitting surface in electon volts.

W_t = kinetic energy of an emitted electron in the "transverse" direction, or parallel to the emitting surface, in electron volts.

W = total emission energy in electron volts.

k = Boltzmann's constant.

T = absolute temperature of emitting surface in degrees Kelvin.

W_T = electron-volt equivalent of kT.

θ = direction of emission velocity relative to the normal to the surface.

$|e|$ = a dimensionless positive constant numerically equal to the charge on the electron.

m = mass of the electron.

J_o = total emission current density in amperes/meter^2.

Some Relationships Between the Above Quantities:

$$W_n = \frac{mu_n^2}{2|e|}; \qquad W_t = \frac{mu_t^2}{2|e|}; \qquad W = W_n + W_t = \frac{mu^2}{2|e|}$$

$$W_T = \frac{kT}{|e|} = \frac{T}{11{,}600} \text{ electron volts.}$$

The Distribution Functions:

The probability that an electron is emitted with a component of velocity normal to the cathode surface in the range u_n to $u_n + du_n$ is

$$dP(u_n) = \frac{mu_n}{kT}\epsilon^{-mu_n^2/2kT}du_n \tag{1}$$

The probability that an electron is emitted with a component of velocity parallel to the cathode surface in the range u_t to $u_t + du_t$ is

$$dP(u_t) = \frac{mu_t}{kT}\epsilon^{-mu_t^2/2kT}du_t \tag{2}$$

The probability that an electron is emitted with kinetic energy normal to the cathode surface in the range W_n to $W_n + dW_n$ is

$$dP(W_n) = \frac{1}{W_T}\epsilon^{-W_n/W_T}dW_n \tag{3}$$

The probability that an electron is emitted with kinetic energy parallel to the cathode surface in the range W_t to $W_t + dW_t$ is

$$dP(W_t) = \frac{1}{W_T}\epsilon^{-W_t/W_T}dW_t \tag{4}$$

The probability that an electron is emitted with total kinetic energy in the range W to $W + dW$ is

$$dP(W) = \frac{W}{W_T}\epsilon^{-W/W_T}\frac{dW}{W_T} \tag{5}$$

The probability that the direction of the emission velocity makes an angle in the range θ to $\theta + d\theta$ with respect to the normal is

$$dP(\theta) = 2\sin\theta\cos\theta d\theta \tag{6}$$

The emission current density per unit solid angle at an angle θ with respect to the normal is

$$J_o\frac{dP(\theta)}{2\pi\sin\theta d\theta} = J_o\frac{\cos\theta}{\pi} \tag{7}$$

APPENDIX V

IN AN AXIALLY SYMMETRIC FIELD THE POTENTIAL AT OFF-AXIS POINTS CAN BE EXPRESSED IN TERMS OF THE POTENTIAL ON THE AXIS AND ITS DERIVATIVES

Here we consider the potential at radius r from an axis of symmetry, which will be designated as the z axis. Laplace's equation for an axially symmetric field is

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{1}$$

Let us suppose that the potential at radius r can be expressed as a power series in r of the form

$$V(z,r) = a_o(z) + a_2(z)r^2 + a_4(z)r^4 + \dots \tag{2}$$

where the odd powers in r are missing because of symmetry about the axis. Substituting this expression into Equation (1), we obtain for the coefficient of r^n

$$(n + 2)^2 a_{n+2} + \frac{d^2 a_n}{dz^2} \tag{3}$$

If the right-hand side of Equation (2) is to be a solution of Equation (1), the coefficient of r^n must be zero. Hence

$$a_{n+2} = -\frac{1}{(n+2)^2}\frac{d^2 a_n}{dz^2} \tag{4}$$

Now $a_o(z)$ is the potential on the axis, or $V(z,0)$. Hence $a_2(z) = -\frac{1}{4}V''(z,0)$, $a_4(z) = +\frac{1}{64}V''''(z,0)$, and so on, where the primes indicate differentiation with respect to z. The potential $V(z,r)$ is therefore given by

$$V(z,r) = V(z,0) - \frac{r^2}{4}V''(z,0) + \frac{r^4}{64}V''''(z,0) - \dots \tag{5}$$

APPENDIX VI

SEVERAL RELATIONS BETWEEN THE OBJECT POSITION, THE IMAGE POSITION AND THE FOCAL LENGTHS OF AN ELECTRON LENS

(a) The Relation $\frac{f_1}{u} + \frac{f_2}{v} = 1$

Figure VI-1 shows three electron trajectories $r_1(z)$, $r_2(z)$, and $r_3(z)$ which pass through a region of axially symmetric field. To the left of the lens the trajectory $r_2(z)$ is parallel to the axis but displaced unit distance from it, whereas to the right of the lens it passes through the focal point F_2. Similarly $r_1(z)$ passes through the focal point F_1 to the left of the lens and emerges parallel to the axis and unit distance from it to the right of the lens. The trajectory $r_3(z)$ crosses the axis u units to the left of the first principal plane and v units to the right of the second principal plane.

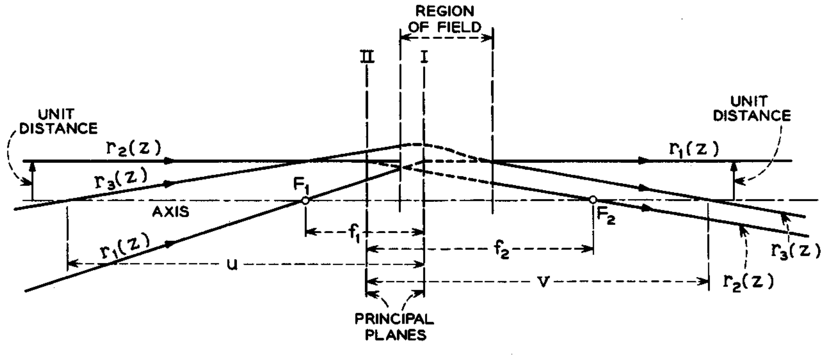

FIG. VI-1 Three electron trajectories which pass through a lens.

Since $r_1(z)$ and $r_2(z)$ are independent solutions of the paraxial ray equation, we can write that

$$r_3(z) = ar_1(z) + br_2(z) \tag{1}$$

where a and b are constants. At the point where $r_3(z)$ crosses the axis to the left of the lens,

$$r_3(z) = 0 = ar_1(z) + br_2(z) = -a\left(\frac{u - f_1}{f_1}\right) + b \tag{2}$$

from which

$$\frac{f_1}{u} = \frac{a}{a + b} \tag{3}$$

At the point where $r_3(z)$ crosses the axis to the right of the lens,

$$r_3(z) = 0 = a - b\left(\frac{v - f_2}{f_2}\right) \tag{4}$$

and

$$\frac{f_2}{v} = \frac{b}{a+b} \tag{5}$$

Adding Equations (3) and (5), we obtain

$$\frac{f_1}{u} + \frac{f_2}{v} = 1 \tag{6}$$

Thus, if we know the focal lengths of a lens and the positions of the principal planes, Equation (6) can be used to determine the focusing action of the lens on any trajectory which is close to the axis and nearly parallel to the axis. In the case of an einzel lens in which the electrodes and potentials are symmetrical about a geometrical mid-point of the lens, $f_1 = f_2 = f$, and Equation (6) becomes

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{7}$$

(b) Magnification $\frac{h_2}{h_1} = \frac{f_1}{f_2}\frac{v}{u}$

With the aid of Figure VI-2 and geometrical considerations similar to those

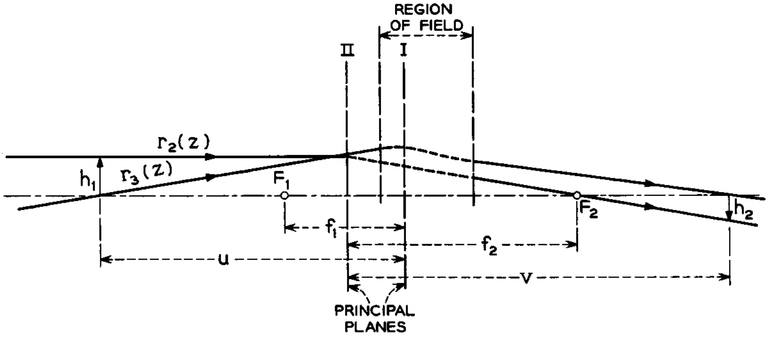

FIG. VI-2 Trajectories used to determine the expression for the magnification of a lens.

used in part (a) above, it is easily shown that the magnification of the lens, given by h_2/h_1, can be expressed as

$$\frac{h_2}{h_1} = \frac{v - f_2}{f_2} = \frac{f_1}{f_2}\frac{v}{u} \tag{8}$$

(c) The Relation $\frac{f_1}{f_2} = \left(\frac{V_1}{V_2}\right)^{1/2}$

Let us suppose that $r_1(z)$ and $r_2(z)$ are two solutions of the paraxial-ray equation, Equation (3.1-8). Then

$$r_1'' + \frac{V'(z,0)}{2V(z,0)}r_1' + \frac{V''(z,0)}{4V(z,0)}r_1 = 0 \tag{9}$$

and

$$r_2'' + \frac{V'(z,0)}{2V(z,0)} r_2' + \frac{V''(z,0)}{4V(z,0)} r_2 = 0 \tag{10}$$

Multiplying the first equation by r_2 and the second by r_1 and subtracting, we obtain

$$(r_1''r_2 - r_2''r_1) + \frac{V'(z,0)}{2V(z,0)}(r_1'r_2 - r_2'r_1) = 0 \tag{11}$$

which can be expressed as

$$\frac{1}{r_1'r_2 - r_2'r_1} \frac{d(r_1'r_2 - r_2'r_1)}{dz} = -\frac{V'(z,0)}{2V(z,0)} \tag{12}$$

Integrating both sides with respect to z, we obtain

$$\ln(r_1'r_2 - r_2'r_1) = -\tfrac{1}{2} \ln V(z,0) + \ln A \tag{13}$$

and hence

$$r_1'r_2 - r_2'r_1 = A[V(z,0)]^{-1/2} \tag{14}$$

where A is a constant. Let us suppose that to the left of the lens $r_2(z) = 1$, $r_2'(z) = 0$, and $V(z,0) = V_1$, and to the right of the lens $r_1(z) = 1$, $r_1'(z) = 0$, and $V(z,0) = V_2$. The trajectories $r_1(z)$ and $r_2(z)$ are therefore as shown in Figure VI-1. From Equation (14), we can write that to the left of the lens

$$r_1' = AV_1^{-1/2} \tag{15}$$

and

$$f_1 = \frac{1}{r_1'} = \frac{V_1^{1/2}}{A} \tag{16}$$

To the right of the lens,

$$-r_2' = AV_2^{-1/2} \tag{17}$$

and

$$f_2 = -\frac{1}{r_2'} = \frac{V_2^{1/2}}{A} \tag{18}$$

Finally, combining Equations (16) and (18), we obtain

$$\frac{f_1}{f_2} = \left(\frac{V_1}{V_2}\right)^{1/2} \tag{19}$$

APPENDIX VII

A STEADY-STATE SOLUTION OF POISSON'S EQUATION FOR A SPACE-CHARGE-LIMITED DIODE IS UNIQUE

The potential in the interelectrode space of a diode satisfies Poisson's Equation,

$$\nabla^2 V(x,y,z) = -\frac{\rho(x,y,z)}{\varepsilon_o} \tag{1}$$

We shall assume that the electrons leave the cathode surface with zero velocity.

Let us first consider the case in which the anode voltage V_{ao} is zero. Clearly, $V(x,y,z) = \rho(x,y,z) = 0$ is the only possible solution in this case. If ρ were not zero, electrons would be present between the electrodes, and the potential in the interelectrode space would be depressed below cathode potential. Field lines then would extend from induced charges on the cathode to the charge in the interelectrode space, and the potential gradient dV/dn at the cathode surface would be negative. However, since the electrons leave the cathode surface with zero velocity, all the electrons would be returned to the cathode immediately upon emission. This contradicts our assumption that there is charge present in the interelectrode space. Thus we must conclude that for $V_{ao} = 0$, $\rho(x,y,z) = 0$. Equation (1) then reduces to Laplace's Equation, which gives $V(x,y,z) = 0$ for the boundary condition $V_{ao} = 0$.

For $V_{ao} > 0$ and for space-charge-limited operation, a steady-state solution to Equation (1) must satisfy the following boundary conditions:

1. $V = \dfrac{dV}{dn} = 0$

at the cathode surface, where d/dn is the derivative in the direction normal to the cathode surface. (Note that the assumption of zero emission velocity means that the potential minimum coincides with the cathode.)

2. $V = V_{ao}$ at the anode surface.

Let us suppose that there are two independent solutions of Equation (1) which meet these boundary conditions for $V_{ao} > 0$. Let the solutions be:

$V_1(x,y,z)$, corresponding to a charge distribution $\rho_1(x,y,z)$

and

$V_2(x,y,z)$, corresponding to a charge distribution $\rho_2(x,y,z)$

In such a case $V = V_1 - V_2$ would be a solution of Equation (1) which satisfies the boundary conditions $V = 0$ at the anode and $V = dV/dn = 0$ at the cathode and which corresponds to a charge distribution $\rho_1 - \rho_2$. But from the discussion in the previous paragraph we know that $V(x,y,z) = \rho(x,y,z) = 0$ is the only solution to Equation (1) which meets the boundary conditions for $V_{ao} = 0$. Thus we conclude that $V_1 = V_2$, and $\rho_1 = \rho_2$. Hence a steady-state solution of Poisson's Equation for a space-charge-limited diode is unique.

APPENDIX VIII

IF A TWO-DIMENSIONAL POTENTIAL IN FREE SPACE IS SYMMETRIC ABOUT AN AXIS, THE POTENTIAL AT OFF-AXIS POINTS CAN BE EXPRESSED IN TERMS OF THE POTENTIAL ON THE AXIS

The derivative of a complex function $f(z) = f(x + jy) = u(x,y) + jv(x,y)$ is defined as

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} \tag{1}$$

where $\Delta z \doteq \Delta x + j\Delta y$. It can be shown that necessary and sufficient conditions for the existence of a unique derivative of $f(z)$ are that

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \tag{2}$$

and

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \tag{3}$$

These equations are known as the Cauchy-Riemann conditions. (See, for instance, R. V. Churchill, *Introduction to Complex Variables and Applications*, McGraw-Hill Book Co., Inc., New York, 1948, p. 30.) A function $f(z) = u + jv$ is said to be *analytic* in a region of the z plane if the derivative $f'(z)$ exists at every point in that region. Examples of analytic functions are z, $z^2 + 1$, e^z, and $\sin z$, where $z = x + jy$ in each case.

If we take the partial derivative of Equation (2) with respect to x and the partial derivative of Equation (3) with respect to y, we obtain

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} \quad \text{and} \quad \frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial^2 u}{\partial y^2} \tag{4}$$

from which

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{5}$$

In a similar manner it can be shown that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \tag{6}$$

Hence the real and imaginary parts of an analytic function of z satisfy Laplace's Equation in two dimensions.

Now $u(x,y)$ can be expressed as

$$u(x,y) = \operatorname{Re} f(x + jy) = \tfrac{1}{2}[f(x + jy) + f(x - jy)] \tag{7}$$

If y in this expression is replaced by $-y$, the value of u is unchanged, so that $u(x,y)$ is symmetric about the x axis. It follows, therefore, that the real part of an analytic function $f(z)$ defines a potential which satisfies Laplace's Equation in two dimensions and which is symmetric about the x axis.

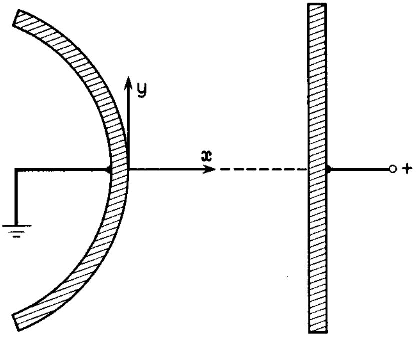

FIG. VIII-1 Two electrodes which are symmetric about an axis and which extend indefinitely above and below the page.

Figure VIII-1 shows two electrodes which are assumed to extend to infinity above and below the page. One is at ground potential, the other is at a positive potential. The coordinate axes shown in the figure are such that the x axis lies in the plane of symmetry of the electrodes. Let us suppose the potential along the x axis between the electrodes is given by $V = f_1(x)$, where $f_1(z) = f_1(x + jy) = u_1(x,y) + jv_1(x,y)$ is an analytic function. Then consider the potential given by

$$u_1(x,y) = \tfrac{1}{2}[f_1(x + jy) + f_1(x - jy)] \quad (8)$$

This satisfies Laplace's Equation in two dimensions, and when $y = 0$, it gives the correct potential along the x axis. Furthermore, it is symmetric about the x axis.

To show that no other function $u_2(x,y)$ also satisfies these conditions, and hence that $u_1(x,y)$ is in fact the potential in the interelectrode space of Figure VIII-1, we can make use of two other results of complex variable theory. These are:

1. Any function $u_2(x,y)$ which satisfies Laplace's Equation in two dimensions defines a function $v_2(x,y)$ such that $f_2(z) = u_2(x,y) + jv_2(x,y)$ is analytic. (See Churchill, p. 139.)

2. If $f_1(z)$ and $f_2(z)$ are analytic throughout a region in the z plane, and if $f_1(z) = f_2(z)$ along a curve within the region, then $f_1(z) = f_2(z)$ throughout the region. (Churchill, page 189.)

It follows from (1) above that the function $u_2(x,y)$ defines an analytic function $f_2(z)$. But $f_2(z) = f_1(z)$ along the x axis, and consequently $f_2(z) = f_1(z)$ at points off the x axis, and $u_2(x,y) = u_1(x,y)$.

APPENDIX IX

APPROXIMATE EXPRESSIONS FOR THE ELECTROSTATIC AMPLIFICATION FACTOR OF A PLANAR TRIODE AND FOR THE FUNCTIONS F_1 *AND* F_2

Here we make use of complex variable theory and that branch of complex variable theory known as conformal mapping to derive approximate expressions for the electrostatic amplification factor of a planar triode and for the functions F_1 and F_2. First it will be helpful to discuss a few concepts relating to conformal mapping.

Suppose that $w = u(x,y) + jv(x,y) = f(z) = f(x + jy)$ is an analytic function of z. (See Appendix VIII for a definition of an analytic function.) The function $f(z)$ "maps" each point z_o in the x-y plane into a corresponding point w_o in the u-v plane. Suppose that two curves C_1 and C_2 in the x-y plane intersect at z_o, and the tangents to these curves at z_o make an angle γ with each other. The two curves will map on the u-v plane into two curves C_1' and C_2' which intersect at w_o. Furthermore it can be shown that, provided $f(z)$ is analytic at z_o and provided $f'(z_o) \neq 0$, the angle between the curves C_1' and C_2' at w_o is also γ. (See, for instance, R. V. Churchill, *Introduction to Complex Variables and Applications*, McGraw-Hill Book Co., Inc., New York, 1948, pp. 135 and 136.) A mapping which preserves angles between pairs of curves in this manner is said to be conformal.

For example, the equation $u(x,y) = a$ defines a curve in the x-y plane, and every point on that curve maps onto a point on the straight line $u = a$ in the u-v plane. (See Figure IX-1.) Similarly, every point on the curve $v(x,y) = b$ in the x-y

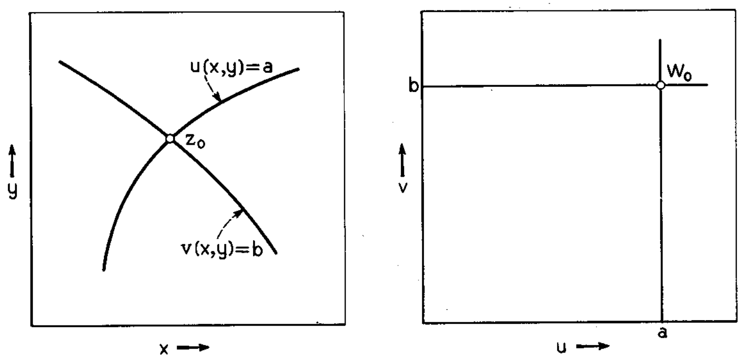

FIG. IX-1 The curves $u(x,y) = a$ and $v(x,y) = b$ in the x-y plane map into the lines $u = a$ and $v = b$ in the u-v plane.

plane maps onto the line $v = b$ in the u-v plane. Suppose the two curves in the x-y plane intersect at z_o. Consider the slope of the curve $u(x,y) = a$ at z_o. Taking the total derivative of $u(x,y) = a$, we obtain

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = 0 \tag{1}$$

and

$$\frac{dy}{dx} = -\frac{\dfrac{\partial u}{\partial x}}{\dfrac{\partial u}{\partial y}} \tag{2}$$

If the partial derivatives in Equation (2) are evaluated at z_o, the expression gives the slope of the curve $u(x,y) = a$ at z_o. Similarly the slope of the curve $v(x,y) = b$ at z_o is given by $-(\partial v/\partial x)/(\partial v/\partial y)$, where the partial derivatives are evaluated at z_o. By applying the Cauchy-Riemann conditions given by Equations (2) and (3) of Appendix VIII, it is easily shown that the slope of one curve is minus the reciprocal of the slope of the second curve and hence that the two curves are orthogonal to each other at z_o. Since the curves map into the lines $u = a$ and $v = b$ in the u-v plane, the mapped curves are likewise orthogonal at the point of intersection, and the mapping preserves the angle between the curves.

Similarly, a set of orthogonal curves giving the equipotential contours and the corresponding field lines for a particular boundary value problem in the x-y plane would map into a second set of orthogonal curves in the u-v plane.

Suppose that $V(x,y)$ is a solution of Laplace's Equation in two dimensions for a particular boundary-value problem in the x-y plane. Then

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \tag{3}$$

Let the potential $V(x,y)$ be generated by applying voltages V_1, V_2, . . . to electrodes 1, 2, . . . in the x-y plane. The curves defining the electrodes in the x-y plane map into corresponding curves in the u-v plane. Let us further suppose that the voltages V_1, V_2, . . . are applied to the "mapped electrodes." Now since each point z_o in the x-y plane defines a point w_o in the u-v plane, the potential $V(x,y)$ can be expressed as $V(u,v)$. Furthermore $V(u,v) = V_1$ at mapped electrode 1, it equals V_2 at mapped electrode 2, and so on. In addition, it is shown below that $V(u,v)$ satisfies Laplace's Equation in the u-v plane, and since solutions to Laplace's Equation are unique, it follows that $V(u,v)$ gives the potential in the region surrounding the mapped electrodes when the voltages V_1, V_2, . . . are applied to them. Thus the equipotential contours and field lines in the x-y plane map onto corresponding equipotential contours and field lines in the u-v plane. This result is useful in solving two-dimensional potential problems because it may be possible to determine $V(u,v)$ more easily than $V(x,y)$. However, once we know $V(u,v)$, we can obtain $V(x,y)$ by a change of variables.

As a final point let us show that $V(u,v)$ satisfies Laplace's Equation in two dimensions. We can write

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial V}{\partial v}\frac{\partial v}{\partial x} \tag{4}$$

and

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial^2 V}{\partial u \partial v}\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial V}{\partial u}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 V}{\partial v^2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{\partial V}{\partial v}\frac{\partial^2 v}{\partial x^2} \tag{5}$$

A similar expression can be obtained for $\partial^2 V/\partial y^2$. Substituting these expressions in

Equation (3) and making use of Equations (2), (3), (5), and (6) of Appendix VIII, we obtain

$$\left(\frac{\partial^2 V}{\partial u^2} + \frac{\partial^2 V}{\partial v^2}\right)\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] = 0 \tag{6}$$

Since the quantity in the rectangular brackets is generally not zero, it follows that

$$\frac{\partial^2 V}{\partial u^2} + \frac{\partial^2 V}{\partial v^2} = 0 \tag{7}$$

Thus $V(u,v)$ satisfies Laplace's Equation.

Next let us see how these results can be applied to a planar triode in the absence of space charge. Figure IX-2(a) shows a portion of the planar triode. One grid

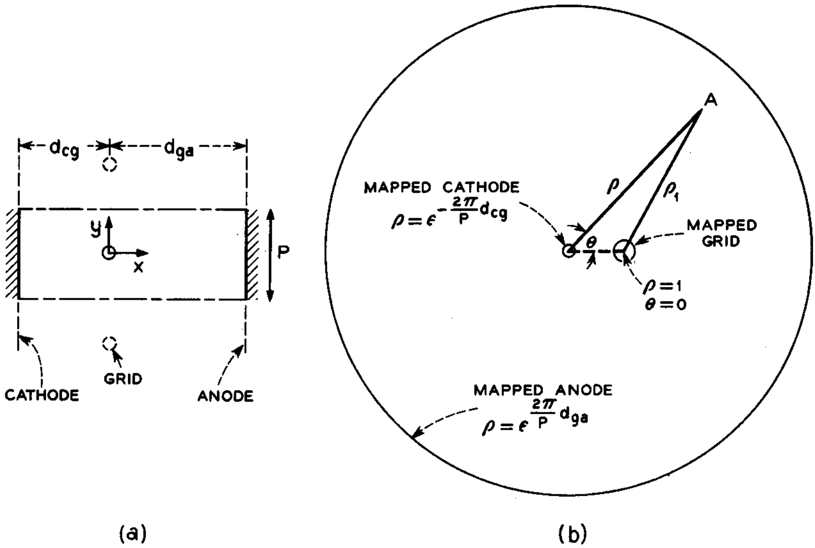

FIG. IX-2 A portion of a planar triode in the x-y plane mapped onto the ρ-θ plane.

wire is shown with a solid ring, and the two adjacent wires are shown with broken rings. Two lines extend from the cathode to the anode midway between the central grid wire and the two adjacent wires. The pitch of the grid is P, so that the spacing between the two lines is P. We shall consider only the portion of the triode between the two lines. Clearly the amplification factor of this portion of the triode will be the same as for the whole device.

It will be convenient to replace $u + jv$ in the foregoing discussion by $\rho\epsilon^{j\theta}$. The transformation which enables us to solve the potential problem in the planar triode is given by

$$u + jv = \rho\epsilon^{j\theta} = f(z) = \epsilon^{2\pi z/P} = \epsilon^{2\pi(x+jy)/P} \tag{8}$$

From this we see that

$$\rho = \epsilon^{2\pi x/P} \tag{9}$$

and

$$\theta = \frac{2\pi}{P}y \tag{10}$$

Figure IX-2(b) shows how the triode maps onto the ρ-θ plane by means of this transformation. The anode maps onto a circle $\epsilon^{2\pi d_{ga}/P}$, where d_{ga} is the grid-to-anode spacing; whereas the cathode maps onto a circle of radius $\epsilon^{-2\pi d_{cg}/P}$, where d_{cg} is the cathode-to-grid spacing. Since $\epsilon^{2\pi} \approx 256$, it is evident that the radius of the mapped anode is very large compared with unity, and the radius of the mapped cathode is very small compared with unity. The center of the grid wire maps onto the point $\rho = 1$, $\theta = 0$, and the perimeter of the grid wire maps onto a closed curve about the point $\rho = 1$, $\theta = 0$. If the radius of the grid wire is very small compared with P, say less than $P/20$, the closed curve is nearly a circle and is small in diameter. Notice that the transformation maps the point $(x, y + nP)$, where n is an integer, onto the same point in the ρ-θ plane as the point (x,y). Consequently the remainder of the triode beyond the part shown in Figure IX-2(a) is mapped on top of the mapping shown Figure IX-2(b).

Suppose the mapped cathode carries an axial charge density $+\tau_c$ coulombs per unit length in the direction normal to the page, and the mapped grid carries an axial charge density $+\tau_g$ coulombs per unit length in the direction normal to the page. We shall assume that the radii of the mapped cathode and mapped grid are sufficiently small that these axial charge densities can be considered as line charges. An expression for the potential resulting from a line charge τ coulombs per unit length surrounded *coaxially* by a cylindrical conductor is obtained by integrating Equation (1.4-5) with respect to radius r. Thus

$$V = V_o - \frac{\tau}{2\pi\varepsilon_o}\ln r \tag{11}$$

where V_o is a constant that adjusts for the level of potential in the region, and r is the radial distance from the line charge to the point where the potential is determined. The potential in the interelectrode space of Figure IX-2(b) is a superposition of that arising from the axial charge density τ_c and that arising from the axial charge density τ_g. The contribution resulting from the axial charge density τ_c is given by Equation (11), where τ becomes τ_c, and r becomes ρ. Since the radius of the mapped anode is much greater than unity, the axial charge density τ_g is *nearly* coaxial with the mapped anode. Hence to a good approximation Equation (11) also can be used to give the potential resulting from this axial charge, where in this case radius r is measured from the point $\rho = 1, \theta = 0$. Thus an approximate expression for the potential at point A is given by

$$\begin{aligned} V(\rho,\theta) &= C - \frac{\tau_c}{2\pi\varepsilon_o}\ln\rho - \frac{\tau_g}{2\pi\varepsilon_o}\ln\rho_1 \\ &= C - \frac{\tau_c}{2\pi\varepsilon_o}\ln\rho - \frac{\tau_g}{4\pi\varepsilon_o}\ln(\rho^2 + 1 - 2\rho\cos\theta) \end{aligned} \tag{12}$$

where $\rho_1 = (\rho^2 + 1 - 2\rho\cos\theta)^{1/2}$ is the distance from the point $\rho = 1$, $\theta = 0$ to point A, and C is a constant which adjusts for the level of potential in the region.

Combining Equations (9), (10), and (12), the potential in the interelectrode space of the planar triode can be expressed as

$$V(x,y) = C - \frac{\tau_c x}{\varepsilon_o P} - \frac{\tau_g}{4\pi\varepsilon_o}\ln\left(\epsilon^{4\pi x/P} + 1 - 2\epsilon^{2\pi x/P}\cos\frac{2\pi}{P}y\right) \tag{13}$$

At the cathode $x = -d_{cg}$, and the exponential terms in the argument of the logarithm are extremely small compared with unity. The argument of the logarithm is therefore nearly 1, and the potential at the cathode is approximately given by

$$V_{co} = C + \frac{\tau_c d_{cg}}{\varepsilon_o P} \tag{14}$$

If we assume that the cathode potential is zero, then

$$C = -\frac{\tau_c d_{cg}}{\varepsilon_o P} \tag{15}$$

At the anode $x = d_{ga}$, and the first term in the argument of the logarithm is far larger than the second and third terms. Neglecting the second and third terms, and substituting for C from Equation (15), the anode potential is found to be

$$V_{ao} = -\frac{\tau_c d_{cg} + \tau_c d_{ga} + \tau_g d_{ga}}{\varepsilon_o P} \tag{16}$$

Examination of the equipotentials given by Equation (13) indicates that in the neighborhood of the origin they are very nearly circles about the origin. If the grid radius R is small compared with P, say $R \leq P/20$, the potential at $(R, 0)$ is therefore very nearly equal to the potential at $(0, R)$. Let us evaluate the potential of the grid by setting $x = 0$ and $y = R$. Then

$$V_{go} = -\frac{\tau_c d_{cg}}{\varepsilon_o P} - \frac{\tau_g}{2\pi\varepsilon_o} \ln 2 \sin \frac{\pi R}{P} \tag{17}$$

where we have made use of the relationship $1 - \cos 2\alpha = 2 \sin^2 \alpha$.

Now the electrostatic amplification factor of the triode is equal to minus the ratio of the anode voltage to grid voltage needed to give zero electric field at the cathode. Since $\tau_c = 0$ when the electric field at the cathode is zero, the electrostatic amplification factor is given by

$$\mu_{es} = -\frac{V_{ao}}{V_{go}}\bigg|_{\text{for } \tau_c=0} = -\frac{2\pi d_{ga}}{P \ln\left(2 \sin \dfrac{\pi R}{P}\right)} \tag{18}$$

As a final point, Equations (16) and (17) can be solved for τ_c and τ_g and the resulting expressions can be substituted in Equation (13). In this way we can express $V(x,y)$ in the form

$$V(x,y) = V_{go}F_1 + V_{ao}F_2 \tag{19}$$

where

$$F_1 = \mu_{es}\frac{d_{cg} + x - (P/4\pi d_{ga})(d_{ga} + d_{cg}) \ln\left(\epsilon^{4\pi x/P} + 1 - 2\epsilon^{2\pi x/P} \cos \dfrac{2\pi}{P} y\right)}{d_{ga} + d_{cg} + \mu_{es} d_{cg}} \tag{20}$$

and

$$F_2 = \frac{d_{cg} + x + (P/4\pi d_{ga})\mu_{es} d_{cg} \ln\left(\epsilon^{4\pi x/P} + 1 - 2\epsilon^{2\pi x/P} \cos \dfrac{2\pi}{P} y\right)}{d_{ga} + d_{cg} + \mu_{es} d_{cg}} \tag{21}$$

At $x = -d_{cg}$, the argument of the logarithmic term is very nearly 1, and the derivative of the logarithmic term is essentially zero. If these approximations are taken into account, it is easily shown that

$$\mu_{es} = \left. \frac{\dfrac{\partial F_1}{\partial x}}{\dfrac{\partial F_2}{\partial x}} \right|_{x=-d_{cg}} \tag{22}$$

Note that the origin for the coordinate system is different here than in Equation (5.1-1).

APPENDIX X

IMPEDANCE OF A SPACE-CHARGE-LIMITED PLANAR DIODE

When a small ac signal is superimposed on the dc voltage applied to a space-charge-limited diode, the electron velocity exhibits an ac component, and an ac induced current flows in the external circuit connected to the diode. Here we derive an expression for the impedance given by the ratio of the applied ac voltage to the ac induced current in the external circuit for the case of a planar diode.

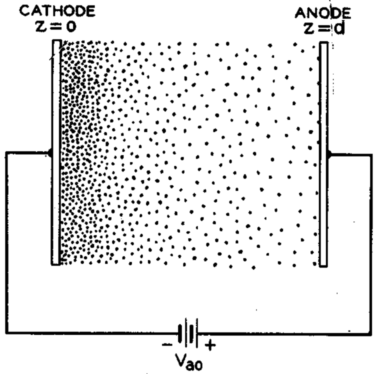

FIG. X-1 A space-charge-limited planar diode.

The planar diode is illustrated in Figure X-1. We assume that the virtual cathode coincides with the actual cathode so that, strictly speaking, our results apply only to this case. Let us gather together three relations which we shall use later in obtaining the impedance of the diode:

1. From Equations (4.1-8) and (4.1-10), the dc current density flowing between the electrodes of the diode is given by

$$J_o = \frac{4\varepsilon_o(2\eta)^{1/2}}{9}\frac{V^{3/2}}{z^2} = \frac{2\varepsilon_o}{9\eta}\frac{u_o^3}{z^2} = \frac{4\varepsilon_o(2\eta)^{1/2}}{9}\frac{V_{ao}^{3/2}}{d^2} \tag{1}$$

where $\eta = e/m$, V is the dc potential at distance z from the cathode, $u_o = \sqrt{2\eta V}$ is the dc electron velocity at z, V_{ao} is the dc anode voltage, and d is the electrode spacing.

2. The electron transit time between the electrodes in the absence of an applied ac signal is given by

$$T_o = \int_0^d \frac{dz}{u} = \left[\frac{6\varepsilon_o d}{\eta J_o}\right]^{1/3} \tag{2}$$

where we have substituted from Equation (1) for u_o.

3. The low-frequency "dynamic anode resistance" for a unit area of the electrodes is given by

$$r_a = \frac{dV_{ao}}{dJ_o} = \frac{2}{3}\frac{V_{ao}}{J_o} = \frac{T_o d}{2\varepsilon_o} \tag{3}$$

Note that this has the dimensions of resistance *times* area.

Let us proceed now with the derivation of the impedance of the diode. Since the fields are one-dimensional, Poisson's Equation for this problem is

$$\frac{\partial E}{\partial z} = \frac{\rho}{\varepsilon_o} \tag{4}$$

where ρ is the space-charge density at the point where E is determined. The equation of continuity, Equation (1.3-2), becomes

$$\frac{\partial \rho(u_o + u)}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \qquad (5)$$

where u is the ac component of the electron velocity. Combining Equations (4) and (5), we obtain

$$\frac{\partial}{\partial z}\left[\rho(u_o + u) + \varepsilon_o \frac{\partial E}{\partial t}\right] = 0 \qquad (6)$$

This equation states that the quantity $\rho(u_o + u) + \varepsilon_o(\partial E/dt)$ is independent of z in the region between the electrodes. The term $\rho(u_o + u)$ is simply the current density resulting from the flow of charge between the electrodes. The second term also has the dimensions of a current density and is called the displacement current density.

To understand the displacement current density better, consider a parallel-plate capacitor in the absence of space charge. If a time-varying voltage V is applied to it, a current $i = C(dV/dt) = \varepsilon_o A(dE/dt)$ flows in the leads to the capacitor. The quantity $\varepsilon_o(dE/dt)$ is the displacement current density. The total displacement current crossing a surface between the plates is equal to the conduction current in the leads. It is a real current in the sense that it gives rise to the same magnetic field that would be produced by a similar distribution of current in a conductor. In the parallel-plate capacitor, the displacement current density is uniform everywhere between the plates. However, in the space-charge-limited diode, the electric field lines extend between charge that is in transit between the electrodes and induced charges on the anode. Clearly E is a function of z in this case, so that the displacement current density also is a function of z.

We shall call the quantity $\rho(u_o + u) + \varepsilon_o(\partial E/dt)$ the total current density and denote it by J_T. The current flowing in the cathode and anode leads of the diode is given by $J_T A$, where A is the area of the electrodes. With the aid of Equation (4), J_T can be expressed as

$$J_T = \rho(u_o + u) + \varepsilon_o \frac{\partial E}{dt} = \varepsilon_o\left(\frac{\partial E}{\partial z}\frac{dz}{dt} + \frac{\partial E}{\partial t}\right) \qquad (7)$$

The quantity in the brackets is the time rate of change of the electric field experienced by a moving electron. The first term arises from the variation of E with z as the electron travels with velocity dz/dt toward the anode, and the second term arises from the time variation of the field. The sum of the terms gives the total derivative of E with respect to time. Hence

$$J_T = \varepsilon_o \frac{dE}{dt} \qquad (8)$$

The acceleration of the electron is given by

$$\frac{d^2 z}{dt^2} = -\eta E \qquad (9)$$

Differentiating this with respect to time and substituting for dE/dt from Equation (8), we obtain

$$\frac{d^3 z}{dt^3} = -\eta \frac{dE}{dt} = -\frac{\eta}{\varepsilon_o} J_T \qquad (10)$$

Equation (6) showed that J_T is independent of z. However, it is not independent of time if the voltage applied to the electrodes is changing with time. If a small sinusoidal voltage is superimposed on the dc voltage between the electrodes, J_T will have a sinusoidal component and can be expressed as

$$J_T = -(J_o + J_1 \sin \omega t) \tag{11}$$

where J_o is the current density given by Equation (1), and J_1 is small compared with J_o. The minus sign implies that the current flows in the minus z direction, or toward the cathode. Substituting in Equation (10) for J_T, we can perform successive integrations with respect to time to obtain expressions for the acceleration, velocity, and position of an electron with respect to time. Let the time the electron left the cathode be t_o. The boundary conditions at time t_o are $z = dz/dt = d^2z/dt^2 = 0$. Integrating Equation (10) from t_o to t then gives

$$\frac{d^2z}{dt^2} = \frac{\eta}{\varepsilon_o}\left[J_o(t - t_o) - \frac{J_1}{\omega}(\cos \omega t - \cos \omega t_o)\right] \tag{12}$$

Integrating a second time gives

$$\frac{dz}{dt} = \frac{\eta}{\varepsilon_o}\left[J_o\frac{(t - t_o)^2}{2} - \frac{J_1}{\omega^2}(\sin \omega t - \sin \omega t_o) + \frac{J_1}{\omega}(t - t_o) \cos \omega t_o\right] \tag{13}$$

Finally,

$$z = \frac{\eta}{\varepsilon_o}\left[J_o\frac{(t - t_o)^3}{6} + \frac{J_1}{\omega^3}(\cos \omega t - \cos \omega t_o) + \frac{J_1}{\omega^2}(t - t_o) \sin \omega t_o + \frac{J_1}{\omega}\frac{(t - t_o)^2}{2} \cos \omega t_o\right] \tag{14}$$

By setting $z = d$ in the last of these equations, the time $t - t_o$ becomes the electron transit time T. If we then multiply both sides of the equation by $6\varepsilon_o/\eta J_o$, the equation can be rewritten in the form $T^3 = T_o^3 + \delta = T_o^3(1 + \delta/T_o^3)$, where δ is a summation of terms containing J_1/J_o as a factor. If the ac voltage applied between the electrodes is small compared with the dc voltage, J_1 is small compared with J_o, and δ/T_o^3 is small compared with unity. We can then write $T = T_o(1 + \delta/3T_o^3) = T_o + \delta/3T_o^2$. Since $\delta/3T_o^2$ is small compared with T_o, we can set $t - t_o = T_o$ and $t_o = t - T_o$ in the expression for $\delta/3T_o^2$. This is equivalent to neglecting terms containing the product of two or more small quantities. Thus we obtain

$$T = T_o - \frac{J_1}{J_o}\frac{2}{\omega^3 T_o^2}\left[\cos \omega t + \omega T_o \sin \omega(t - T_o) + \left(\frac{(\omega T_o)^2}{2} - 1\right)\cos \omega(t - T_o)\right] \tag{15}$$

As a final part of our calculations we shall integrate the electric field E from the cathode to the anode to obtain an expression for the instantaneous voltage between the cathode and anode. Thus

$$v_a = -\int_o^d E dz = \frac{1}{\eta}\int_o^d \frac{d^2z}{dt^2}dz = \frac{1}{\eta}\int_{t_o=t}^{t_o=t-T} \frac{d^2z}{dt^2}\frac{dz}{dt_o}dt_o \tag{16}$$

where d^2z/dt^2 is the acceleration of an electron at distance z from the cathode at time t, and we have substituted $E = -(1/\eta)d^2z/dt^2$ from Equation (9). The derivative dz/dt_o can be found by differentiating Equation (14) with respect to t_o. Thus

$$\frac{dz}{dt_o} = -\frac{\eta}{\varepsilon_o}\frac{(t - t_o)^2}{2}(J_o + J_1 \sin \omega t_o) \tag{17}$$

We can now substitute from Equations (12) and (17) into Equation (16) and carry out the integration. We shall neglect terms containing the product of two or more small quantities, that is, terms containing J_1^2 and higher powers of J_1. The time T appears in the answer, and we can substitute for it from Equation (15). Thus we obtain

$$v_a = V_o + \frac{\eta J_o J_1}{\varepsilon_o^2 \omega^4}\left\{[2(1 - \cos \omega T_o) - \omega T_o \sin \omega T_o] \sin \omega t - \left[\frac{(\omega T_o)^3}{6} + \omega T_o(1 + \cos \omega T_o) - 2 \sin \omega T_o\right] \cos \omega t\right\} \tag{18}$$

This is of the form

$$v_a = V_o + rJ_1 \sin \omega t + xJ_1 \cos \omega t \tag{19}$$

where r and x are, respectively, a resistance for a unit area and a reactance for a unit area and are given by

$$r = 12r_a\left[\frac{2(1 - \cos \omega T_o) - \omega T_o \sin \omega T_o}{(\omega T_o)^4}\right] = r_a\left[1 - \frac{(\omega T_o)^2}{15} + \dots\right] \tag{20}$$

$$x = -12r_a\left[\frac{1}{6\omega T_o} + \frac{\omega T_o(1 + \cos \omega T_o) - 2 \sin \omega T_o}{(\omega T_o)^4}\right] = -r_a\left[\frac{3}{10}\omega T_o - \frac{(\omega T_o)^3}{84} + \dots\right] \tag{21}$$

and $r_a = \frac{2}{3}V_{ao}/J_o$ from Equation (3). The quantities r/r_a and x/r_a are plotted in Figure 7.1-6 as functions of ωT_o.

An ac induced current $J_1 A \sin \omega t$ flows in the external circuit in response to the applied ac voltage given by Equation (19), where A is the area of the electrodes. The diode therefore presents an impedance given by $Z = (r/A) + j(x/A)$ to the applied ac voltage. At low frequencies, or small ωT_o, this impedance reduces to

$$Z = \frac{r_a}{A}\left(1 - j\frac{3}{10}\omega T_o\right) \tag{22}$$

In constructing an equivalent network for the device it is more useful to use the admittance given by the reciprocal of this, or

$$Y = \frac{1}{Z} \approx \frac{A}{r_a}\left(1 + j\frac{3}{10}\omega T_o\right) = g_o + j\omega\frac{3}{5}C_o \tag{22}$$

where $g_o = A/r_a = A\partial I_{ao}/\partial V_{ao}$, and $C_o = \varepsilon_o A/d$ is the capacitance of the parallel-plate capacitor formed by the anode and cathode in the absence of space charge. This equation indicates that at low frequencies the diode acts as a conductance $g_o = A\partial J_o/\partial V_{ao}$ shunted by a capacitance equal to $\frac{3}{5}$ times the capacitance of the diode in the absence of space charge. A low-frequency equivalent network for the diode is shown in Figure 7.1-8.

APPENDIX XI

LLEWELLYN-PETERSON COEFFICIENTS

Tabulated below are the coefficients in the Llewellyn-Peterson equations discussed in Section 7.2.

$$A^* = \frac{1}{\varepsilon_o}(u_{ao} + u_{bo})\frac{T^2}{2}\frac{1}{\beta}\left[1 - \frac{\zeta}{3}\left(1 - \frac{12S}{\beta^3}\right)\right]$$

$$B^* = \frac{1}{\varepsilon_o}\frac{T^2}{\beta^3}\left[u_{ao}(P - \beta Q) - u_{bo}P + \zeta(u_{ao} + u_{bo})P\right]$$

$$C^* = -\frac{1}{\eta}2\zeta(u_{ao} + u_{bo})\frac{P}{\beta^2}$$

$$D^* = 2\zeta\left(\frac{u_{ao} + u_{bo}}{u_{bo}}\right)\frac{P}{\beta^2}$$

$$E^* = \frac{1}{u_{bo}}[u_{bo} - \zeta(u_{ao} + u_{bo})]\epsilon^{-\beta}$$

$$F^* = \frac{\varepsilon_o}{\eta}\frac{2\zeta}{T^2}\left(\frac{u_{ao} + u_{bo}}{u_{bo}}\right)\beta\epsilon^{-\beta}$$

$$G^* = -\frac{\eta}{\varepsilon_o}\frac{T^2}{\beta^3}\frac{1}{u_{bo}}[u_{bo}(P - \beta Q) - u_{ao}P + \zeta(u_{ao} + u_{bo})P]$$

$$H^* = -\frac{\eta}{\varepsilon_o}\frac{T^2}{2}\left(\frac{u_{ao} + u_{bo}}{u_{bo}}\right)(1 - \zeta)\frac{\epsilon^{-\beta}}{\beta}$$

$$I^* = \frac{1}{u_{bo}}[u_{ao} - \zeta(u_{ao} + u_{bo})]\epsilon^{-\beta}$$

The following dc quantities and relationships pertain to these coefficients:

u_{ao} and u_{bo} are the dc electron velocities at planes a and b.

T is the electron transit time from plane a to plane b in the presence of the dc space charge in the interelectrode space but in the absence of applied ac signals.

T_o is the electron transit time from plane a to plane b in the absence of space charge and in the absence of applied ac signals. (Note that T and T_o have meanings in this appendix different from those in Appendix X.)

ζ is a space charge factor related to T and T_o by $\zeta = 3(1 - T_o/T)$.

If d is the distance from plane a to plane b, then

$$d = (1 - \zeta/3)\,(u_{ao} + u_{bo})\frac{T}{2}$$

If J_o is the dc current density passing through either of the planes,

$$J_o = \frac{\varepsilon_o}{\eta}(u_{ao} + u_{bo})\frac{2\zeta}{T^2}$$

The following quantities contain the angular frequency ω of the ac signal.

$\beta = j\omega T$, where $j = \sqrt{-1}$, and ω is 2π times the frequency of the ac signal.

$$P = 1 - \epsilon^{-\beta} - \beta\epsilon^{-\beta} \approx \frac{\beta^2}{2} - \frac{\beta^3}{3} + \frac{\beta^4}{8} \cdots$$

$$Q = 1 - \epsilon^{\beta} \approx \beta - \frac{\beta^2}{2} + \frac{\beta^3}{6} - \frac{\beta^4}{24} \cdots$$

$$S = 2 - 2\epsilon^{-\beta} - \beta - \beta\epsilon^{-\beta} \approx -\frac{\beta^3}{6} + \frac{\beta^4}{12} - \frac{\beta^5}{40} + \frac{\beta^6}{180} \cdots$$

APPENDIX XII

SOME USEFUL VECTOR RELATIONSHIPS

Let **A** be an arbitrary vector and Φ an arbitrary scalar in the real or phasor notation. $\mathbf{i}_x$, $\mathbf{i}_y$, etc. are unit vectors in the coordinate directions indicated by the subscripts. The following relationships apply to **A** and Φ. In rectangular coordinates:

$$\mathbf{A} = \mathbf{i}_x A_x + \mathbf{i}_y A_y + \mathbf{i}_z A_z \tag{1}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{2}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{i}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{i}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \tag{3}$$

$$\nabla\Phi = \mathbf{i}_x\frac{\partial \Phi}{\partial x} + \mathbf{i}_y\frac{\partial \Phi}{\partial y} + \mathbf{i}_z\frac{\partial \Phi}{\partial z} \tag{4}$$

In cylindrical coordinates:

$$\mathbf{A} = \mathbf{i}_r A_r + \mathbf{i}_\theta A_\theta + \mathbf{i}_z A_z \tag{5}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_r}{\partial r} + \frac{A_r}{r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \tag{6}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_r\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right) + \mathbf{i}_\theta\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{i}_z\left(\frac{\partial A_\theta}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta} + \frac{A_\theta}{r}\right) \tag{7}$$

$$\nabla\Phi = \mathbf{i}_r\frac{\partial \Phi}{\partial r} + \mathbf{i}_\theta\frac{1}{r}\frac{\partial \Phi}{\partial \theta} + \mathbf{i}_z\frac{\partial \Phi}{\partial z} \tag{8}$$

In any coordinate system, the following two theorems are true:

Gauss's Theorem

$$\int_{\text{closed surface}} \mathbf{A}\cdot\mathbf{n}\,dS = \int_{\text{volume}} \nabla\cdot\mathbf{A}\,dv \tag{9}$$

Stoke's Theorem

$$\oint_{\text{closed loop}} \mathbf{A}\cdot d\mathbf{l} = \int_{\text{surface}} (\nabla \times \mathbf{A})\cdot\mathbf{n}\,dS \tag{10}$$

The following vector identities may be useful:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B}) \tag{11}$$

$$\nabla\cdot(\mathbf{A} \times \mathbf{B}) = \mathbf{B}\cdot(\nabla \times \mathbf{A}) - \mathbf{A}\cdot(\nabla \times \mathbf{B}) \tag{12}$$

$$\nabla^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}) \tag{13}$$

Equation (13) defines the Laplacian of a vector. In *rectangular* coordinates it is given by

$$\nabla^2\mathbf{A} = \mathbf{i}_x\nabla^2 A_x + \mathbf{i}_y\nabla^2 A_y + \mathbf{i}_z\nabla^2 A_z \tag{14}$$

where

$$\nabla^2 A_x = \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \tag{15}$$

$$\nabla^2 A_y = \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \tag{16}$$

$$\nabla^2 A_z = \frac{\partial^2 A_z}{\partial x^2} \quad \frac{\partial^2 A_z}{\partial y^2} \quad \frac{\partial^2 A_z}{\partial z^2} \tag{17}$$

Detailed proofs of the above relationships are given in texts on vector analysis or advanced calculus. A good discussion is given in J. B. Hildebrand, *Advanced Calculus for Engineers,* Chapter 6, Prentice-Hall, Inc., New York, 1949.

APPENDIX XIII

GROUP VELOCITY AND ENERGY FLOW

The physical significance of the group velocity can be made clearer by considering the propagation of a pulse modulated carrier wave down a lossless transmission line or waveguide of arbitrary length L.[1] This line is assumed to propagate the energy in a single mode, which can be characterized by one branch of an ω-β diagram of the type shown in Figure 8.5-2.

Let us suppose that some information in the form of a pulse of electromagnetic energy is impressed into the waveguide at the input end. We can ask ourselves the question: How much time will elapse before we can detect this pulse or information at the output end, a distance L away?

Since all of the field components in a single mode are related by Maxwell's Equations, we can study the propagation of just one of them, and this will characterize the behavior of the other components as well. Assume that this

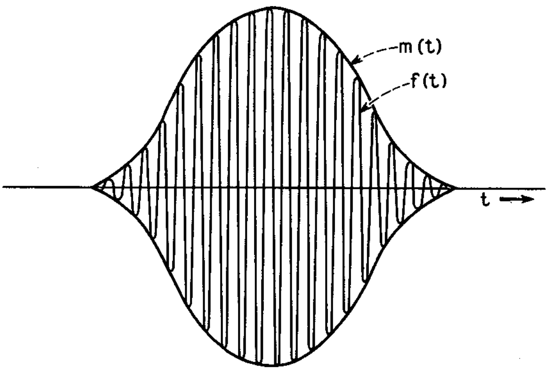

FIG. XIII-1 A high-frequency carrier with a modulation envelope $m(t)$.

component has a time variation $f(t)$ at the input as shown in Figure XIII-1. As is usual with modulation systems, the rate of change of the envelope $m(t)$ is assumed to be very slow compared with the frequency of the carrier. This means that a Fourier analysis of the resultant waveform $f(t)$ would yield frequency components clustered closely to the carrier frequency. Thus $f(t)$ may be written as

$$f(t) = \operatorname{Re} m(t)\epsilon^{j\omega_o t} \tag{1}$$

where ω_o is the carrier radian frequency.

The modulation envelope $m(t)$ may be written in terms of its Fourier transform

$$m(t) = \int_{-\infty}^{\infty} M(p)\epsilon^{jpt}dp \tag{2}$$

[1]Reference 8h, pp. 81–84.

Since any practical modulation system will have a finite bandwidth, this equation can be written

$$m(t) = \int_{-p_o}^{p_o} M(p)\epsilon^{jpt}dp \tag{3}$$

This expression may be used in Equation (1), obtaining

$$f(t) = \text{Re}\int_{-p_o}^{p_o} M(p)\epsilon^{j(\omega_o+p)t}dp \tag{4}$$

The quantity $f(t)$ is the superposition of terms of slightly different frequencies, centered about the carrier frequency. Thus, the pulse of energy is a superposition of waves of different frequencies, each with its own propagation constant. Each component has a different phase shift from the input to the output. Since the system is linear and lossless, the output is a superposition of the input frequencies, each one shifted in phase by the proper amount. If $g(t)$ is the time variation of the field component at the output,

$$g(t) = \text{Re}\int_{-p_o}^{p_o} M(p)\epsilon^{j(\omega_o+p)t}\epsilon^{-j\beta L}dp \tag{5}$$

where β is a function of $\omega_o + p$, the frequency. Since all of the frequency components are close to the carrier frequency, the variation of the propagation constant with frequency may be adequately represented by the first term of a Taylor series.

$$\beta = \beta_o + \frac{\partial\beta}{\partial\omega}p \tag{6}$$

where β_o is the propagation constant for the carrier frequency. We thus obtain

$$g(t) = \text{Re}\,\epsilon^{j(\omega_o t-\beta_o L)}\int_{-p_o}^{p_o} M(p)\epsilon^{jp[t-(\partial\beta/\partial\omega)L]}dp \tag{7}$$

This represents a carrier modulated by a modulation envelope $n(t)$, where

$$n(t) = \int_{-p_o}^{p_o} M(p)\epsilon^{jp[t-(\partial\beta/\partial\omega)L]}dp \tag{8}$$

Comparing with Equation (3), we see that

$$m(t) = n\left(t + \frac{\partial\beta}{\partial\omega}L\right) \tag{9}$$

that is, the modulation envelope at the output is exactly reproduced from the input, but at a time $(\partial\beta/\partial\omega)L$ later. This is just as if the pulse of energy had traveled with a velocity

$$v_g = \frac{\partial\omega}{\partial\beta} \tag{10}$$

This is the physical significance of the group velocity.

The same relationship holds for all of the field components. Hence, if we could visualize the electromagnetic bundle of energy, we would see it move physically with the velocity v_g.

These results may be applied to continuous wave propagation at a single frequency. That is, we may think of a continuous wave as being a superposition of rectangular modulated pulses placed end-to-end. It is clear that the power flow in this case is given by

$$P = v_g W_l \tag{11}$$

where W_l is the total energy stored per unit length.

APPENDIX XIV

TIME AVERAGE STORED ENERGY

The instantaneous magnetic stored energy is given by

$$\mathcal{W}(t) = \tfrac{1}{2}\mu_o \int_{\text{volume}} |\mathcal{H}(t)|^2 dv \tag{1}$$

The instantaneous magnetic field may be written in the phasor notation

$$\mathcal{H}(t) = \text{Re}\mathbf{H}\epsilon^{j\omega t} \tag{2}$$

where

$$\mathbf{H} = \mathbf{H}_r + j\mathbf{H}_i$$

and $\mathbf{H}_r$ and $\mathbf{H}_i$ are real. Thus

$$\mathcal{H}(t) = \mathbf{H}_r \cos \omega t - \mathbf{H}_i \sin \omega t \tag{3}$$

Equation (1) may be written

$$\mathcal{W}(t) = \tfrac{1}{2}\mu_o \int_{\text{volume}} (|\mathbf{H}_r|^2 \cos^2 \omega t + |\mathbf{H}_i|^2 \sin^2 \omega t - 2\mathbf{H}_r \cdot \mathbf{H}_i \sin \omega t \cos \omega t) dv \tag{4}$$

The time average of this quantity is given by

$$W_{\text{avg}} = \tfrac{1}{4}\mu_o \int_{\text{volume}} [|\mathbf{H}_r|^2 + |\mathbf{H}_i|^2] dv \tag{5}$$

But this is simply

$$W_{\text{avg}} = \tfrac{1}{4}\mu_o \int_{\text{volume}} |\mathbf{H}|^2 dv \tag{6}$$

Now in a periodic structure, since the time average magnetic stored energy equals the time average electric stored energy, the total time average stored energy per cell is given by

$$W_L = \tfrac{1}{2}\mu_o \int_{\text{unit cell}} |\mathbf{H}|^2 dv \tag{7}$$

APPENDIX XV

KLYSTRON INTERACTION FOR A HIGH DEGREE OF BUNCHING

In the theory of klystron interaction, large values of the bunching parameter ($X > 1$) result in crossings of the electron trajectories such that a given arrival time near the bunch center corresponds to three different departure times. This behavior is illustrated in Figure 9.1-2.

In Figure XV-1, the curve for $X = 1.5$ is replotted as τ_b vs. τ_a, where

$$\tau_a = \omega t_o \tag{1}$$

and

$$\tau_b = \omega t - \theta \tag{2}$$

That is, τ_a and τ_b are normalized departure and arrival times, respectively. Certain

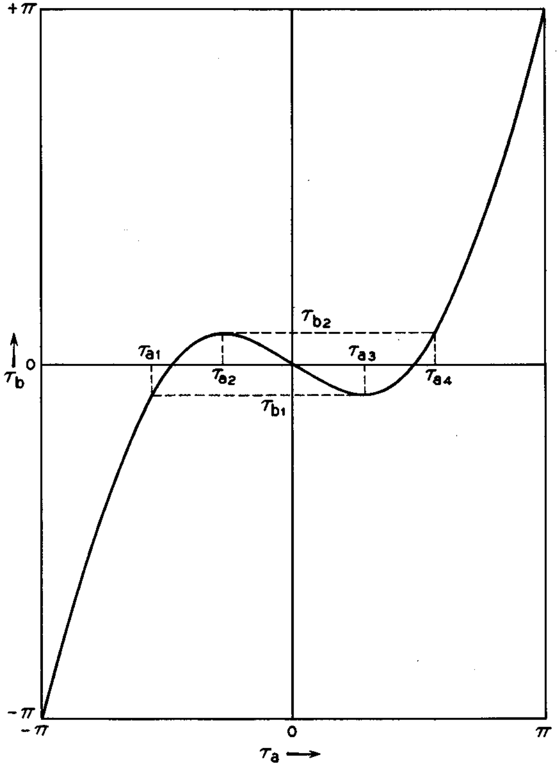

FIG. XV-1 Output gap arrival time in radians plotted vs. the input gap departure time in radians for $X = 1.5$.

points have been labeled on this graph which characterize the limits of the multivalued branches.

Assuming that the subscript 2 in Equation (9.1-19) refers to the branch with negative slope from τ_{a2} to τ_{a3}, Equation (9.1-19) may be written as

$$i(t) = -I_o\left[\frac{d\tau_a}{d\tau_b}\bigg|_1 - \frac{d\tau_a}{d\tau_b}\bigg|_2 + \frac{d\tau_a}{d\tau_b}\bigg|_3\right] \tag{3}$$

Equation (9.1-20) for the Fourier coefficient a_n becomes

$$\begin{aligned} a_n = &-\frac{I_o}{\pi}\int_{-\pi}^{\tau_{b1}} \frac{d\tau_a}{d\tau_b}\cos n\tau_b d\tau_b \\ &-\frac{I_o}{\pi}\int_{\tau_{b1}}^{\tau_{b2}}\left[\frac{d\tau_a}{d\tau_b}\bigg|_1 - \frac{d\tau_a}{d\tau_b}\bigg|_2 + \frac{d\tau_a}{d\tau_b}\bigg|_3\right]\cos n\tau_b d\tau_b \\ &-\frac{I_o}{\pi}\int_{\tau_{b2}}^{\pi}\frac{d\tau_a}{d\tau_b}\cos n\tau_b d\tau_b \end{aligned} \tag{4}$$

Consider the second integral term. When the variable of integration is changed, one has:

$$\int_{\tau_{b1}}^{\tau_{b2}} \frac{d\tau_a}{d\tau_b}\bigg|_1 \cos n\tau_b d\tau_b = \int_{\tau_{a1}}^{\tau_{a2}} \cos n\tau_b d\tau_a \tag{5}$$

$$\int_{\tau_{b1}}^{\tau_{b2}} -\frac{d\tau_a}{d\tau_b}\bigg|_2 \cos n\tau_b d\tau_b = \int_{\tau_{a3}}^{\tau_{a2}} -\cos n\tau_b d\tau_a = \int_{\tau_{a2}}^{\tau_{a3}} \cos n\tau_b d\tau_a \tag{6}$$

and

$$\int_{\tau_{b1}}^{\tau_{b2}} \frac{d\tau_a}{d\tau_b}\bigg|_3 \cos n\tau_b d\tau_b = \int_{\tau_{a3}}^{\tau_{a4}} \cos n\tau_b d\tau_a \tag{7}$$

When Equations (5), (6), and (7) are added together, one obtains the simple result:

$$\int_{\tau_{a1}}^{\tau_{a4}} \cos n\tau_b d\tau_a \tag{8}$$

Equation (4) thus becomes simply:

$$a_n = -\frac{I_o}{\pi}\int_{-\pi}^{\pi} \cos n\tau_b d\tau_a \tag{9}$$

This equation is the same as the first of Equations (9.1-22) which was derived for the case of small bunching parameter ($X < 1$), and hence it leads to the same Fourier coefficients. The b_n coefficients may be shown to be identical in the same fashion.

APPENDIX XVI

A DERIVATION OF THE EXPRESSION FOR THE THERMAL NOISE GENERATED BY A RESISTANCE

Here we derive an expression for the thermal noise generated by a resistance, using as a basis for our derivation the random motions of the charge carriers in the

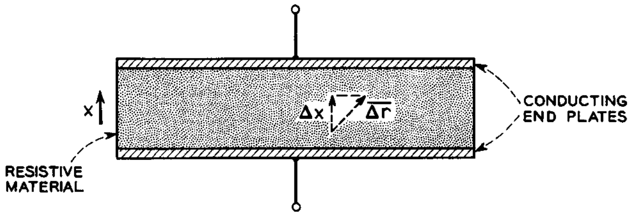

FIG. XVI-1 A cylinder of resistive material with two conducting end plates.

resistance.[1] Figure XVI-1 shows a view of the resistance. We shall assume that the resistance consists of a cylinder of resistive material between two conducting end plates. The end plates are joined by an external wire. We shall further assume that the charge carriers within the resistance have a charge $-e$, a density n per unit volume, a mobility μ, and a diffusion constant D. (See Section 14.2 for a discussion of diffusion and mobility.) If a voltage v were applied across the resistance, there would be an electric field v/L within the resistance, where L is the distance from one end plate to the other, and a current

$$i = \frac{v}{L} e\mu nA = \frac{v}{R} \tag{1}$$

would flow through the resistance, where A is the area of the end plates, and R is the magnitude of the resistance. We assume that the linear dimensions of the end plates are sufficiently large in comparison with L that the effects of fringing fields can be neglected. The resistance R is then given by

$$R = \frac{L}{e\mu nA} \tag{2}$$

If there is no applied voltage across the resistance, the motions of the charge carriers is of a random-walk nature, sometimes called Brownian motion. (For a discussion of Brownian motion, see E. H. Kennard, *Kinetic Theory of Gases*, Sections 160–164, McGraw-Hill Book Co., Inc., New York, 1938. A second informative discussion is given by D. K. C. MacDonald, *Noise and Fluctuations: An Introduction*, Section 1.2, John Wiley and Sons, Inc., New York 1962.) On a microscopic scale the individual charge carriers travel along highly irregular paths which are char-

[1]This derivation follows that given by K. M. Van Vliet and J. Blok, *Physica* **22**, 231, 1956.

acterized by frequent abrupt changes in direction. For such motion, it can be shown that the mean squared displacement per carrier in one coordinate direction, say the x direction, in time t is given by

$$\overline{\Delta x^2} = 2Dt. \tag{3}$$

A derivation of this equation is given in *Kinetic Theory of Gases*, Section 163.

In Figure XVI-1 let the x direction be parallel to the axis of the cylindrical resistance. If an individual carrier with charge $-e$ travels a distance Δr within the resistance, where Δr has a component Δx in the x direction, there is a net displacement of charge

$$\Delta q = e\frac{\Delta x}{L} \tag{4}$$

in the external circuit, where again it is assumed that the linear dimensions of the end plates are large compared with the distance L. This equation is similar to Equation (6.1-7). Squaring both sides of this equation and taking the mean value for a large number of carriers, we obtain

$$\overline{\Delta q^2} = e^2\frac{\overline{\Delta x^2}}{L^2} \tag{5}$$

Combining this with Equation (3), *the mean squared displacement of charge in the external circuit per carrier in time* Δt is

$$\overline{\Delta q(\Delta t)^2} = \frac{e^2 2D\Delta t}{L^2} \tag{6}$$

Now the diffusion coefficient is related to the mobility by

$$D = \frac{kT}{e}\mu \tag{7}$$

as in Equation (14.2-12). Combining Equations (2), (6), and (7), we obtain

$$\overline{\Delta q(\Delta t)^2} = \frac{2kT\Delta t}{N_o R} \tag{8}$$

where $N_o = nAL$ is the total number of carriers within the resistance.

If we set $i_e(\Delta t) = \frac{\Delta q(\Delta t)}{\Delta t}$, where $i_e(\Delta t)$ is the mean current in time Δt resulting from the charge displacement $\Delta q(\Delta t)$, we have

$$\overline{i_e(\Delta t)^2} = \frac{2kT}{N_o R\Delta t} \tag{9}$$

The subscript e is used here to indicate that we are dealing with the motion of an individual carrier. This expression gives the mean of the square of the average current flowing in time Δt in the external wire as a result of the motion of an individual carrier within the resistance. By Fourier analysis it can be shown[2] that the

[2]C. J. Bakker, *Physica* **5**, 581, July, 1938. See Equation (17).

frequency distribution associated with a rapidly fluctuating current with mean square average value over a time Δt of $\overline{i_e(\Delta t)^2}$ is given by

$$\overline{i_e^2} = 2\overline{i_e(\Delta t)^2}\Delta t \Delta f \tag{10}$$

where Δf is the bandwidth over which $\overline{i_e(t)^2}$ is measured. Substituting from Equation (9), we obtain

$$\overline{i_e^2} = \frac{4kT}{N_o R}\Delta f \tag{11}$$

Since the motions of the individual carriers are independent of one another, the mean-square current flowing in the external wire as a result of the motions of all N_o carriers is N_o times that given by Equation (11) and hence is given by

$$\overline{i^2} = \frac{4kT}{R}\Delta f \tag{12}$$

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